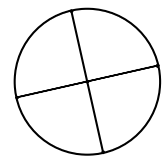
Create an “Arc Measuring Tool”

1. You should have two sheets of Patty Paper. On each sheet construct a large circle. Be sure your circles are congruent to each other.

2. Cut out each circle and set one aside.

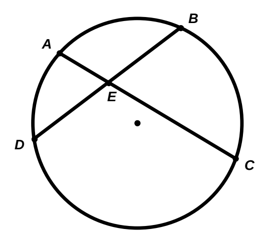
3. Fold a diameter in the second circle. Unfold the circle, then fold a second diameter perpendicular to the first diameter. You should have something that looks like this.



4. What special point is the point of intersection of the diameters? How do you know?

5. You now have a tool to estimate the number of degrees in arcs of your other circle. How can you make your “Arc Measuring Tool” a more precise measuring tool?

6. In your second circle, use a straight edge to draw two chords that intersect at a point that is not the center of the circle. Label your diagram as shown. Then use your available tools to find or estimate the necessary measures to complete the table below.



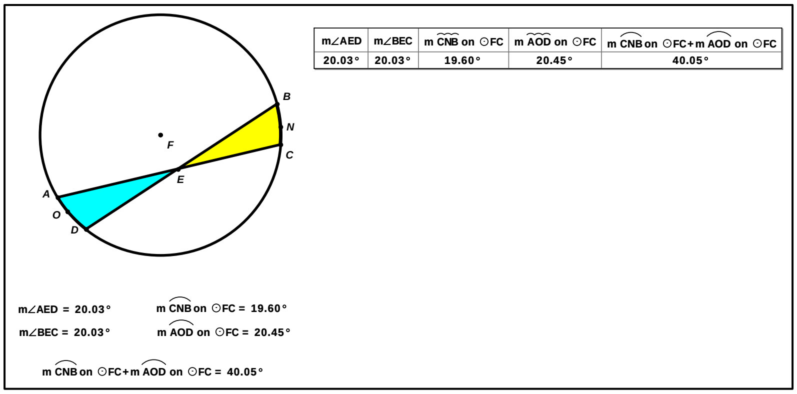
7. Record your name, your measurements and the name of each member of your group along with their measurements in the table.

Name	$m\angle AED$	$m\angle BEC$	$m\overset{\frown}{BC}$	$m\overset{\frown}{AD}$

8. What patterns do you observe in the table?

Angles Formed by Chords Intersecting Inside a Circle

Open the sketch **Twochords-in.**



1. Double click on the table to add another row, then click and drag point *B* away from point *N*. What do you observe?

2. Double click on the table again, and then move point *C* away from point *N*. Be sure point *N* stays between *B* and *C*.
3. Double click again, but this time drag point A away from point *O*. Double click again and drag point *D* away from point *O*. Be sure point *O* stays between *A* and *D*.
4. Be sure you have some small angle measures that are greater than 0° and some large angle measures that are less than 180°. Repeat this process until you have 10 rows in your table.
5. Record the data from the computer in the table below.

$m\angle AED$	$m\angle BEC$	$m\widehat{BC}$	$m\widehat{AD}$	$m\widehat{CNB} + m\widehat{AOD}$

6. What patterns do you observe in the table?

7. To explore the relationship between the sum of the measures of the intercepted arcs and the measure of $\angle AED$, transfer the necessary data from the table in question 3 to the table below.

$m\angle AED$ (x)	PROCESS	$m\overset{\frown}{CNB} + m\overset{\frown}{AOD}$ (y)
x		y

8. Use the process column to develop an algebraic rule that describes this relationship.

9. Write a verbal description of the relationship between the sum of the measures of the intercepted arcs and the measure of the angle formed by the intersecting chords.

10. Create a scatterplot of sum of the arc measures versus angle measure. Describe your viewing window and sketch your graph.

x-min =
x-max =
y-min =
y-max =

11. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

12. Does the graph verify your function rule? Why or why not?

13. What is the measure of an angle formed by two intersecting chords if the measures of its intercepted arcs are 30° and 120°?

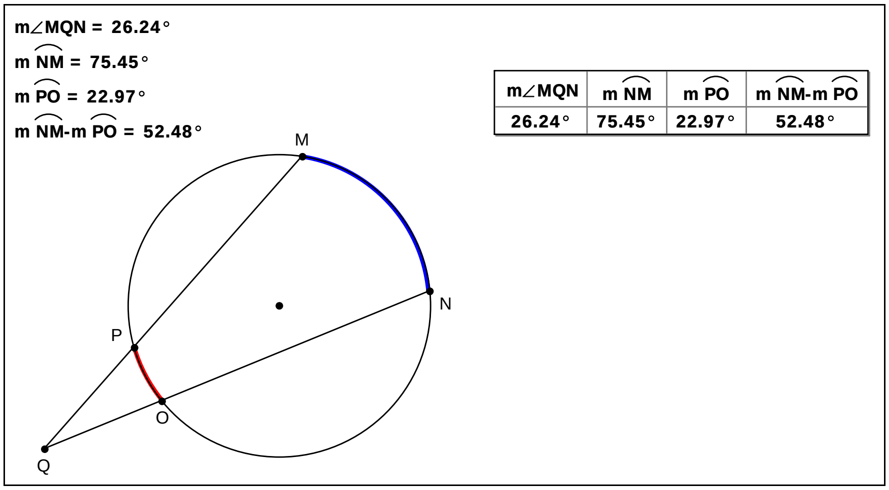
14. What is the sum of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting chords is 56°?

15. Make a general statement about how you can determine the measure of an angle formed by two intersecting chords when you know the measures of the intercepted arcs.

16. Make a general statement about how you can determine the sum of the measures of the intercepted arcs when you know the measure of the angle formed by two intersecting chords.

Angles Formed by Secants Intersecting Outside a Circle

Open the sketch **Twosecant-out**.



1. Double click on the table to add another row, then click and drag point *M*. What do you observe?

2. Double click on the table to add another row, and then move point *M* again. Double click again, but this time drag point N being careful not to drag any point past, or on top of any other point. Repeat this process to add rows to your table.

3. You will need 10 rows of data. Be sure you have some small angle measures and some large angle measures. The angle measures should be greater than 0° and less than 90°.

4. Record the data from the computer in the table below.

$m\angle MQN$	$m\overset{\frown}{MN}$	$m\overset{\frown}{PO}$	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$

5. What patterns do you observe in the table?

6. To explore the relationship between the difference of the measures of the intercepted arcs and the measure of $\angle MQN$, transfer the necessary data from the table in question 4 to the table below.

$m\angle MQN$ (x)	PROCESS	$m\overset{\frown}{MN} - m\overset{\frown}{PO}$ (y)
x		y

7. Use the process column to develop an algebraic rule that describes this relationship.

8. Write a verbal description of the relationship between the difference of the measures of the intercepted arcs and the measure of the angle formed by the intersecting secants.

9. Create a scatterplot of difference of the arc measures vs. angle measure. Describe your viewing window

x-min =
x-max =
y-min =
y-max =

10. Enter your function rule into your graphing calculator and graph your rule over your data. Sketch your graph.

11. Does the graph verify your function rule? Why or why not?

12. What is the measure of an angle formed by two intersecting secants if the measures of its intercepted arcs are 40° and 130°?

13. What is the difference of the measures of the two intercepted arcs if the measure of the angle formed by the intersecting secants is 43°?

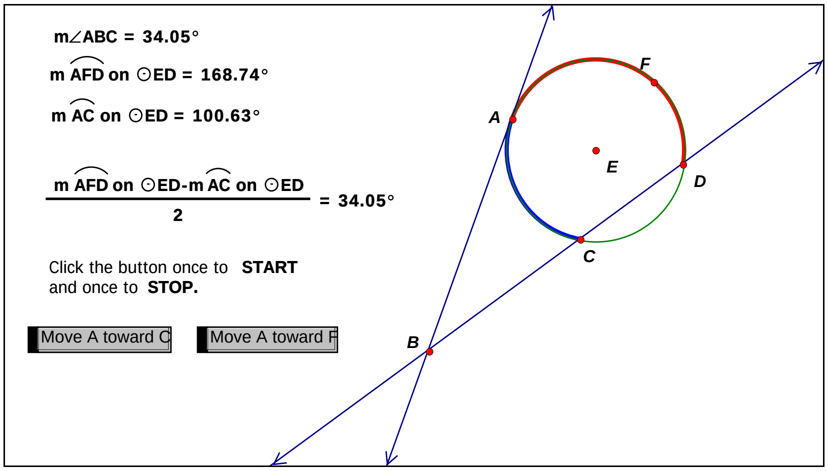
14. Make a general statement about how you can determine the measure of the angle when you know the measures of the intercepted arcs.

15. Make a general statement about how you can determine the difference of the measures of the intercepted arcs when you know the measure of the angle.

Other Intersecting Lines and Segments

1. Tangent and a Secant that intersect in the exterior of a circle

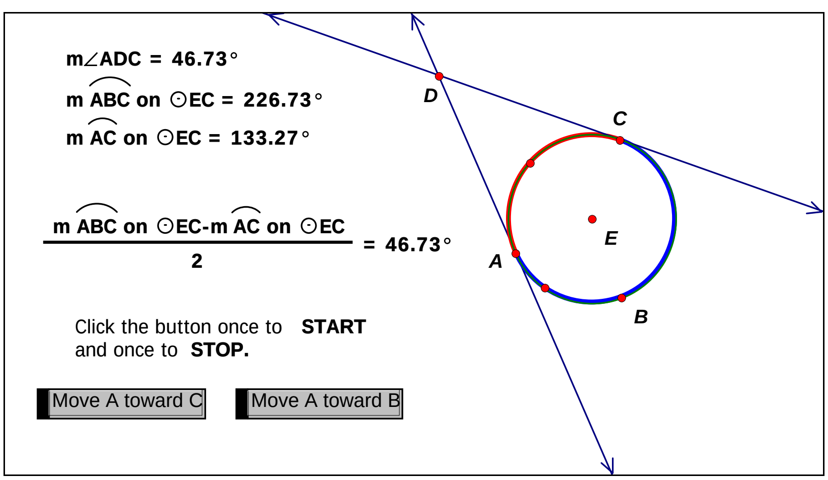
   a. Open the sketch, "**Tansecant-out.**"



   b. Click a button to move point A. What do you observe about the angle and arc relationships?

2. Two tangents that intersect in the exterior of a circle

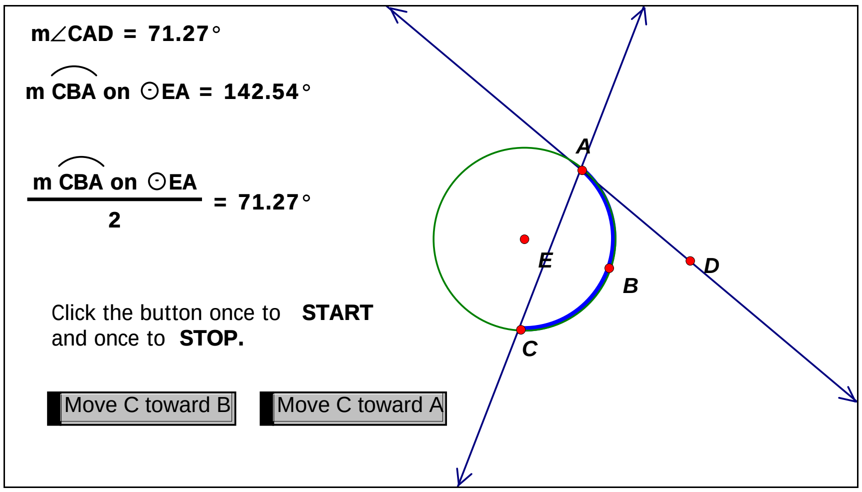
   a. Open the sketch, "**Twotangents-out.**"



   b. Click a button to move point A. What do you observe about the angle and arc relationships?

3. Tangent and a Secant that intersect on a circle

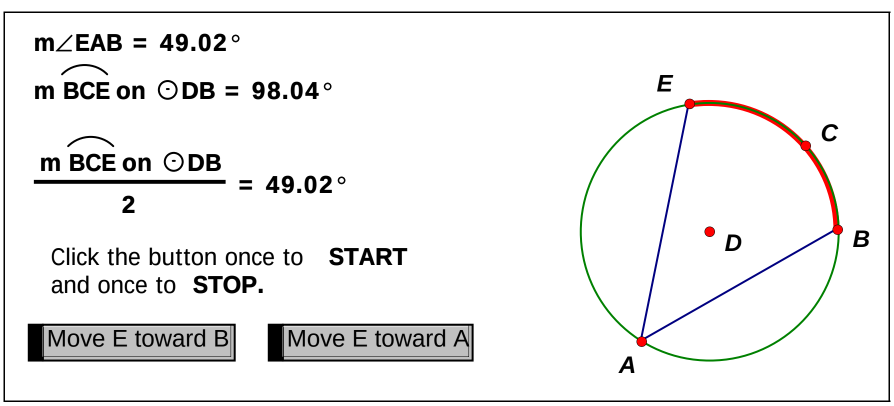
   a. Open the sketch "**Tansecant-on.**"



   b. Click a button to move point C. What do you observe about the angle and arc relationships?

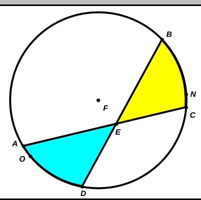
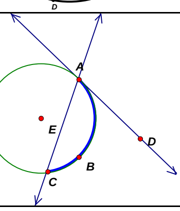
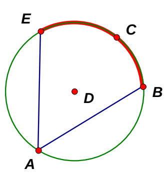
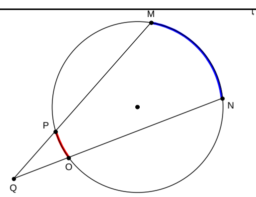
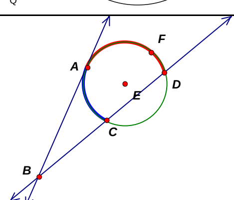
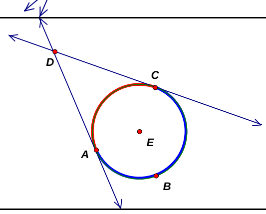
4. Two chords that intersect on a circle

   a. Open the sketch "**Twochords-on.**"



   b. Click a button to move point E. What do you observe about the angle and arc relationships?

In the previous activities you investigated relationships among circles, arcs, chords, secants, and tangents. The vertex of the angle formed by the intersecting lines was either inside the circle, outside the circle or on the circle. Use what you discovered to complete the table below.

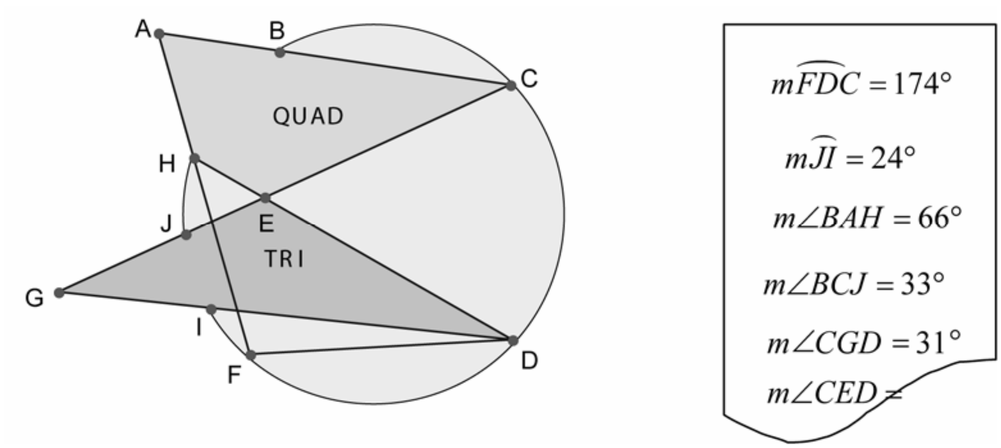
Diagram	Is the vertex of the angle inside, outside or on the circle?	How to calculate the measure of the angle
		
		
		
		
		
		

Complete the following generalizations about calculating angle measure.

1. When the vertex is **inside** the circle, ______ the measures of the intercepted arcs then _______________.
2. When the vertex is **outside** the circle, ______ the measures of the intercepted arcs then _______________.
3. When the vertex is **on** the circle,___________________________.

Quad-Tri Incorporated

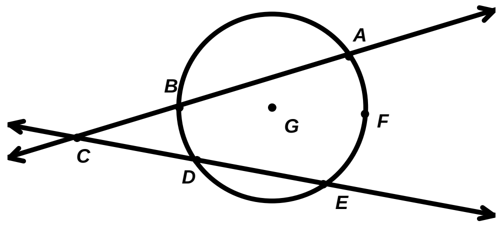
The owners of Quad-Tri Inc. were in the process of designing a new emblem for their employee uniforms when a hurricane rolled in. After the hurricane, Pierre, the chief designer, could only find a torn sheet of paper that contained some of the measures he needed to complete the emblem. The design and the sheet of paper are shown below.



Pierre thinks the measure of angle *CED* must be 60°. Is he correct? Justify your answer.

Circles, Angle Measures and Arcs

1 In the diagram $m\angle BCD = 25°$ and $m\overset{\frown}{BD} = 33°$.



Find $m\overset{\frown}{AFE}$.

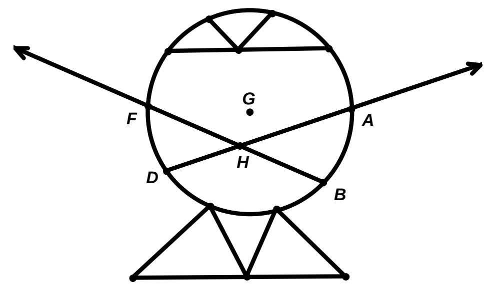
A 17°

B 50°

C 58°

D 83°

2 The metal sculpture shown was found in a recent archeological dig. $m\overset{\frown}{AB} = 46°$ and $m\overset{\frown}{FD} = 38°$



What is $m\angle DHB$?

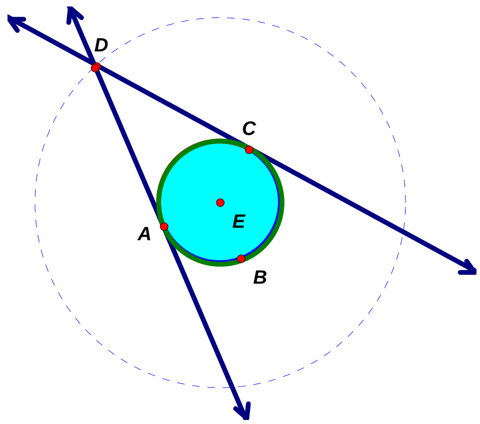
A 4°

B 42°

C 84°

D 138°

3 In the diagram, Point *D* represents a spacecraft as it orbits the Earth.



At this location 220° of the Earths surface is not visible from the spacecraft. What must be the $m\angle ADC$?

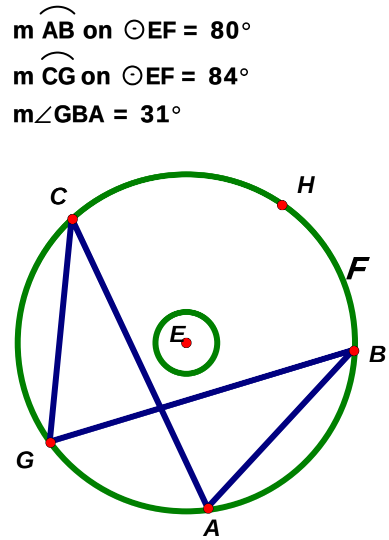
A 40°

B 80°

C 110°

D 140°

4 Pablo created the sketch below.



Based on the measurements he took, what must be $m\overset{\frown}{CHB}$?

A 134°

B 82°

C 67°

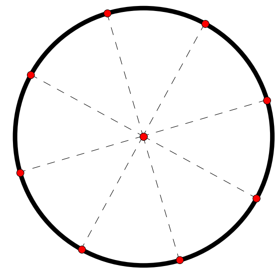
D 33.5°

Area of Regular Polygons

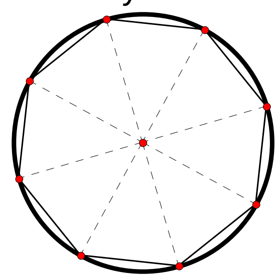
1. On a sheet of patty paper construct a large circle.

2. Cut out the circle.

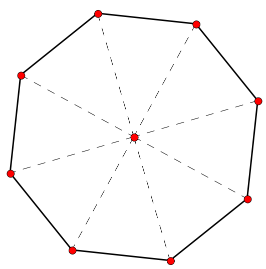
3. Use paper folding to divide the circle into 8 congruent sectors.



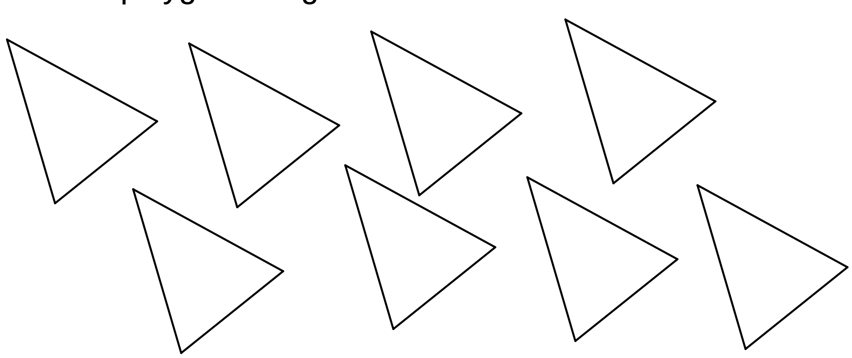
4. Use a straight edge to connect the endpoints of the radii you folded.



5. Cut out the polygon.



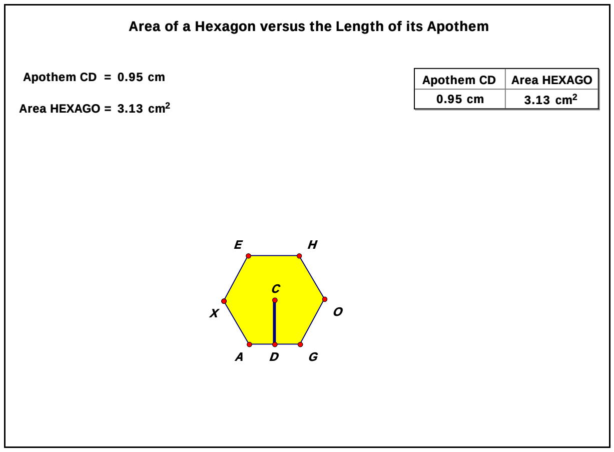
6. Cut the polygon along each fold.



7. Determine the area of your original polygon.

Area of a Regular Hexagon versus the Length of its Apothem

Open the sketch **HEXAGO.**



1. Double click on the table to add another row, then click and drag point *G* a short distance to the right. What do you observe?

2. Double click on the table again, then move point *G* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.

3. Record the data from the computer into the table below.

Apothem CD	*Area HEXAGO*

4. What patterns do you observe in the table?

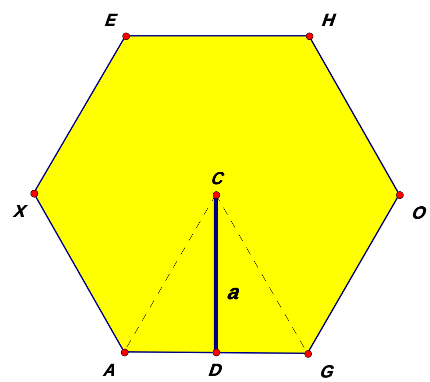
5. What is a reasonable domain and range for your data?

6. Create a scatterplot of Area of a Regular Hexagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.

x-min =
x-max =
y-min =
y-max =

7. What observations can you make about your graph?

8. To help develop a function rule for this situation use Hexagon *HEXAGO* to complete the following.



a. Since *HEXAGO* is a regular hexagon, $m\angle ACG = 60°$. What is $m\angle ACD$?

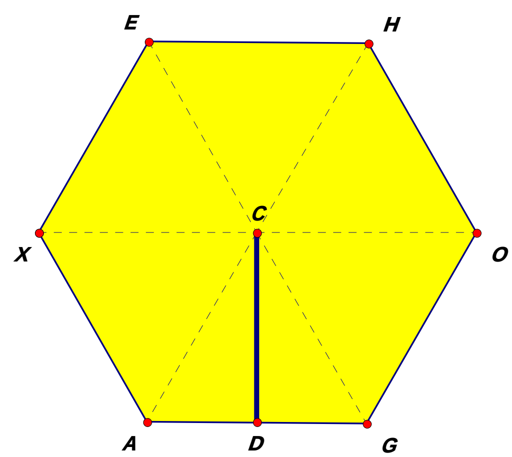
b. Using $\angle ACD$ as the reference angle, the trigonometric ratio "tangent" can be used to find AD in terms of the apothem length, a.

$$tan\,30° = \frac{AD}{a} \text{ or } AD = a(tan\,30°)$$

c. Write an expression for AG in terms of a and tan 30°.

d. Recall the formula for area of a triangle, $Area = \frac{bh}{2}$. Using the length of the apothem a and your answer to question (c) above, write and simplify an expression for the area of ΔACG.

e. Draw the radius to each vertex of Hexagon *HEXAGO*. How many congruent isosceles triangles are formed?



f. Use your answer to questions (d) and (e) above to write an expression for the area of a hexagon.

g. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.

9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

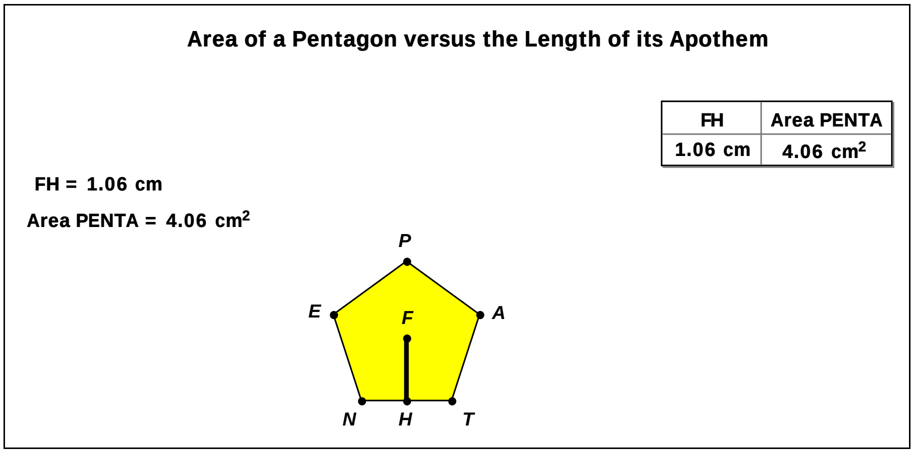
10. Does the graph verify your function rule? Why or why not?

11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular hexagon with an apothem of 6.5 centimeters. Sketch your graph and table.

12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular hexagon with an area of 235.78 square centimeters. Sketch your graph and table.

Area of a Regular Pentagon versus the Length of its Apothem

Open the sketch **PENTA.**



1. Double click on the table to add another row, then click and drag point *T* a short distance to the right. What do you observe?

2. Double click on the table again, and then move point *T* a little farther to the right. Repeat this process until you have 10 rows in your table. Keep the range of the apothem values between 0 and 12.

3. Record the data from the computer in the table below.

Apothem FH	*Area PENTA*

4. What patterns do you observe in the table?

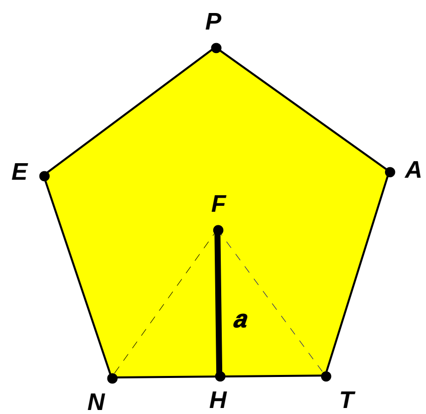
5. What is a reasonable domain and range for your data?

6. Create a scatterplot of Area of a Regular Pentagon versus the Length of its Apothem. Describe your viewing window and sketch your graph.

   x-min =
   x-max =
   y-min =
   y-max =

7. What observations can you make about your graph?

8. To help develop a function rule for this situation, use Pentagon *PENTA* to complete the following.



   a. Since *PENTA* is a regular pentagon, what is $m\angle NFH$?

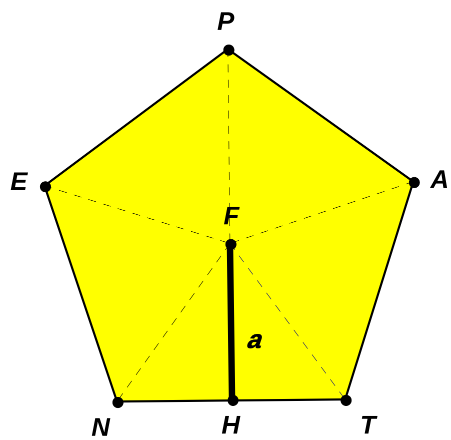
   b. Using $\angle NFH$ as the reference angle, the trigonometric ratio, tangent, can be used to find *NH* in terms of the apothem length, *a*.

   c. Complete the expression *NH =* ________________.

   d. Write an expression for *NT* in terms of, *a, and* tan 36°.

e. Recall the formula for area of a triangle, $Area = \frac{bh}{2}$. Using the length of the apothem, a, and your answer to question (c) above, write and simplify an expression for the area of ΔNFT .

f. Draw the radius to each vertex of Pentagon *PENTA.* How many congruent isosceles triangles are formed?



g. Use your answer to questions d and e above to write an expression for the area of a regular pentagon.

h. Write your expression as a function rule that can be entered into the function graph tool of your graphing calculator.

9. Enter your function rule into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

10. Does the graph verify your function rule? Why or why not?

11. Use your function rule and the graph and table features of your graphing calculator to determine the approximate area of a regular pentagon with an apothem of 8.5 centimeters. Sketch your graph and table.

12. Use your function rule and the graph and table features of your graphing calculator to determine the approximate length of the apothem of a regular pentagon with an area of 400.51 square centimeters. Sketch your graph and table.

Equilateral Triangles and Regular Octagons

In the previous investigations you developed two function rules.

To determine the area, *y*, of a regular **hexagon** given the length of its apothem, *a*, the function rule is:

$$y = 6x^2(tan(30))$$

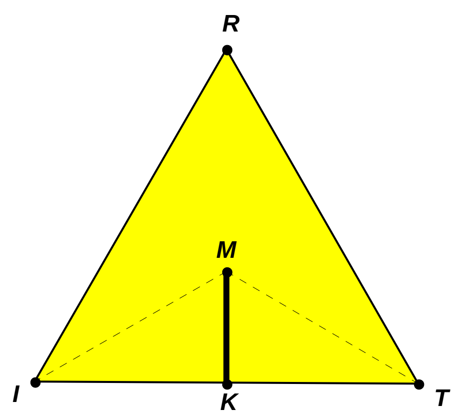
To determine the area, *y*, of a regular **pentagon** given the length of its apothem, *a*, the function rule is:

$$y = 5x^2(tan(36))$$

1. How are the function rules alike? What accounts for the similarities?

2. How are the function rules different? What accounts for the differences?

3. Examine ΔTRI. What is $m\angle TMI$? What is $m\angle IMK$?



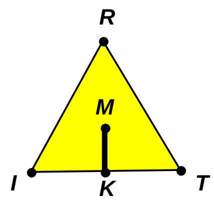
4. Based on your answers to questions 1, 2 and 3 above, write what you think will be the function rule to determine the area, *y*, of a regular **triangle** (equilateral) given the length of its apothem, *a*.

5. Open the sketch, "**TRI.**"

Area of an Equilateral Triangle versus the Length of its Apothem

Apothem MK = 0.62 cm
Area △TRI = 2.01 cm^2

Apothem MK	Area △TRI
0.62 cm	2.01 cm^2



6. Click and drag point *T*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.

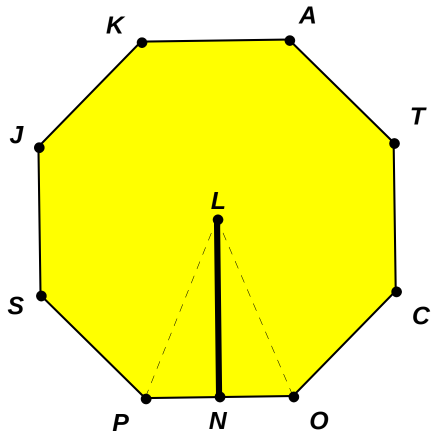
Apothem MK	*Area Triangle TRI*

7. Create a scatterplot of Area of $\triangle TRI$ versus Apothem *MK.*

8. Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

9. Does the graph verify your function rule? Why or why not?

10. Examine regular octagon *OCTAKJSP*. What is $m\angle PLO$? What is $m\angle PLN$?



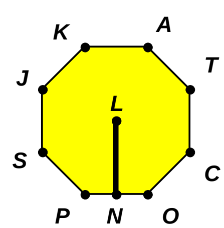
11. Write what you think will be the function rule to determine the area, *y*, of a regular **octagon** given the length of its apothem, *a*.

12. Open the sketch, "**OCTAGONS.**"

Area of a Regular Octagon versus the Length of its Apothem

Apothem LN = 1.06 cm

Area Octagon = 3.73 cm^2



Apothem LN	Area Octagon
1.06 cm	3.73 cm^2

13. Click and drag point *O*. Double click on the table. Continue this process until you have at least 10 data points. Record your data in the table below.

Apothem LN	*Area Octagon*

14. Create a scatterplot of Area of the Octagon versus Apothem *LN.*

15. Enter your function rule from question 4 into your graphing calculator and graph your rule over your scatterplot. Sketch your graph.

16. Does the graph verify your function rule? Why or why not?

17. In the previous activities you investigated relationship between area of regular polygons and the length of their apothems. The table below includes function rules for triangles, pentagons, hexagons, and octagons. Fill in any missing information, then develop a general function rule that can be used to find the area of any regular polygon.

Regular Polygon	Number of Sides	Measure of the Central Angle	Function Rule
Triangle	*3*	*120°*	$y = 3x^2(\tan(60))$
Square			
Pentagon	*5*	*72°*	$y = 5x^2(\tan(36))$
Hexagon	*6*	*60°*	$y = 6x^2(\tan(30))$
Heptagon			
Octagon	*8*	*45°*	$y = 8x^2(\tan(22.5))$
Any	*n*		

18. Use words to describe how to calculate the area of any regular polygon when you know the length of its apothem.

Kick It Incorporated

Banish's company, "Kick It Incorporated," manufactures soccer balls. To construct the covering for each ball 20 regular hexagons and 12 regular pentagons cut from synthetic leather are sewn together. The length of the apothem of each hexagon is 1.5 inches, and the length of the apothem of each pentagon is 1.2 inches.

The shipping manager needs to ship six inflated balls to a customer. He has a box with dimensions 22 inches by 15 inches by 8 inches. Can he fit 2 rows of 3 balls in the box? Justify your answer.

Composite Area

1 The floor of a room is in the shape of a regular hexagon. If the area of the room is 200 square feet, what is the approximate length of the apothem of the hexagon?

A 4.39 feet

B 7.60 feet

C 8.78 feet

D 38.11 feet

3 The table below was generated by a function rule that calculates the area of a regular polygon (y) given the length of its apothem (x).

X	Y1	
1	3.6327	
1.5	8.1736	
2	14.531	
2.5	22.704	
3	32.694	
3.5	44.501	
4	58.123	

X=1

Which polygon was it?

A triangle

B pentagon

C octagon

D decagon

2 The length of the apothem of the STOP sign at the corner of Ashcroft Drive and Ludington Street is 12 inches.

What is the area of the STOP sign?

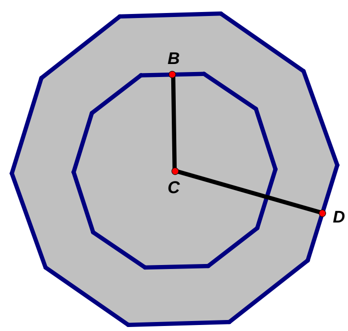
A 96

B 144 square inches

C 477.17

D 498.83

4 The drawing shows a cement walkway around a swimming pool. The walkway and the pool are in the shape of regular polygons. The length of $\overline{BC}$ is 18 feet and the length of $\overline{CD}$ is 29 feet.



What is the area of the walkway?

A 1052.7 square feet

B 2732.6 square feet

C 1873.2 square feet

D 1679.9 square feet